Astro 501, Fundamental Astronomy | Problem Set 6
Due Nov 20

1. A globular cluster has the following properties:

Property	Value
Distance	7 kpc
Core Radius, r_0	0.6 arcmin
Central Radial Velocity Dispersion	7 kms s^{-1}
Central Surface Brightness	16.5 mag arcsec^{-2}

What is the mass-to-light ratio of the cluster? Give your results in solar units, i.e., solar masses per solar V-band luminosity. (To translate magnitudes to solar luminosities, you can use the table at the end of the notes on stellar temperature.)

Angular size can be changed to distance easily using $r = D \sin\theta$. At a distance of 7 kpc, the scale is $s = 0.034$ pc/arcsec, so a core radius of 0.6 arcmin corresponds linear scale of 1.22 pc. For isothermal spheres, the central density is related to r_0 and the central velocity dispersion by

$$r_0 = \left(\frac{9\sigma^2}{4\pi G\rho_0}\right)^{1/2} \implies \rho_0 = \left(\frac{9\sigma^2}{4\pi G r_0^2}\right)$$

Plugging in the numbers gives a value for the central density of $\rho = 3.70 \times 10^{-19}$ g cm^{-3} = 5470 $M_\odot$ pc^{-3}.

The apparent central surface brightness of the cluster is $\mu(V) = 16.5$ mag arcsec^{-2}. We can translate this apparent magnitude to absolute magnitude by

$$\mu_0(M_V) = \mu_0(V) - (5\log D - 5)) = 2.27$$

and then to solar luminosities by

$$\mu_0(M_V) = -2.5\log S_0(V) + C \implies \log S_0(V) = 1.02$$

where $C = 4.83$ is the absolute V-band luminosity of the Sun, and $S_0(V)$ is the luminosity surface brightness at the center of the cluster in solar luminosities per square arcsec. This can then easily be changed into solar luminosities per square parsec by using the distance and dividing by the scale squared, i.e., $I_0(V) = S_0(V)/s^2 = 9135\ L_\odot$ pc^{-2}.

Now note that this value is a luminosity per area, not a luminosity per volume. In order to compare with the mass density, you need to derive the central luminosity density, j_0. This can be done by noting that for the center of an isothermal sphere, the central surface luminosity (or mass) density is related to the central 3-D luminosity (or mass) density by

$$\frac{I_0}{j_0 r_0} \approx 2$$

(Actually, the precise number is 2.018.) So $j_0 \approx I_0/2r_0 \approx 3700 L_\odot$ pc^{-3}. Thus the mass-to-light ratio of the cluster is $1.47 M_\odot/L_\odot$.

2. According to Kennicutt (1988), the total star formation rate of a galaxy can be estimated from the ultraviolet luminosity of its young stars, with the conversion,

$$\mathrm{SFR} = 1.4 \times 10^{-28} L_\nu$$

where the star formation rate is in $M_\odot$ yr^{-1} and the UV luminosity density, L_ν, is in ergs s^{-1} Hz^{-1}. A recent galaxy survey with the GALEX satellite in the UV band found that in the nearby universe, the luminosity function of galaxies can be fit with a Schechter function with $M^*(AB) = -18.04$, $\alpha = -1.22$, and $\phi^* = 0.00426$ galaxies Mpc^{-3}. Assume that all this ultraviolet light comes from young stars. What is the global star formation rate density of the universe today?

The definition of an AB magnitude is

$$M_{AB} = -2.5 \log F_\nu - 48.60$$

where F_ν is in ergs cm^{-2} s^{-1} Hz^{-1}. Therefore, an absolute magnitude of $M^*_{AB} = -18.04$ corresponds to a flux density (at 10 pc) of 5.97×10^{-13} ergs cm^{-2} s^{-1} Hz^{-1}. The total luminosity density of an $M^*_{AB} = -18.04$ galaxy is then

$$L_\nu = F \times 4\pi(10 \text{ pc}^2) = 7.14 \times 10^{27} \text{ ergs s}^{-1} \text{ Hz}^{-1}$$

We can now use the Schechter function to calculate the total luminosity density of the universe:

$$\begin{aligned}\int_0^\infty \phi(L) L \, dL &= \int_0^\infty \phi^* L \, (L/L^*)^\alpha \, e^{-L/L^*} d(L/L^*) \\ &= \phi^* L^* \Gamma(\alpha + 2) = 3.61 \times 10^{25} \text{ ergs s}^{-1} \text{ Hz}^{-1}\end{aligned}$$

If we multiply this number by the Kennicutt conversion, then the star formation rate density of the universe is $0.00506 M_\odot$ yr^{-1} Mpc^{-3}.

3. In his classic 1958 paper, de Vaucouleur describes the Andromeda galaxy as having two components: an exponential disk, with a scale length of 26 arcmin, and a B-band central surface brightness of $B(0) = 21.7$ magnitudes arcsec^{-2}, and an $r^{1/4}$-law bulge with an effective radius of $R_e = 17.5$ arcmin and a B-band surface brightness at R_e of $B_e = 22.86$ magnitudes per arcsec^2. Assume that the galaxy is at a distance of 750 kpc, and has $E(B-V) = 0.062$ foreground extinction, with $R_B = 4.1$. What is the total absolute B-magnitude of M31's disk? What is the total absolute magnitude of M31's bulge? Which component is brighter?

The first task is to convert the surface brightness unit of magnitudes per square arcsec to the linear unit of flux per square arcsec. This is done simply through

$$m = -2.5 \log I + C$$

where m is the magnitude, C is a constant, and I has units of ergs cm^{-2} s^{-1} Hz^{-1} arcsec^{-2}. To get the total flux, one then integrates the disk (or bulge) component over the entire length of the galaxy, from zero to infinity. To be perfectly correct for M31, one needs to know the inclination of the galaxy, but for simplicity, let's just assume the galaxy is face-on. In that case, the integral is

$$F = \int_0^\infty 2\pi r I(r) dr$$

Once the total flux is found, the units can be converted back to magnitudes using the equation above. This will be the total apparent magnitude, uncorrected for extinction. You then apply the extinction correction, which, for $R_B = 4.1$ and $E(B-V) = 0.062$ is

$$m_0 = m_{\text{obs}} - R_B E(B-V) = m_{\text{obs}} - 0.254$$

and convert to absolute magnitude using

$$M_0 = m_0 - 5 \log D + 5 = m_0 - 24.375$$

where $D = 750$ kpc.

The integral for the disk is easy

$$L_{tot} = \int_0^\infty 2\pi r I(r) dr = 2\pi I(0) \int_0^\infty e^{-r/r_0} r dr = 2\pi r_0^2 \cdot I(0)$$

For $r_0 = 26$ arcmin $= 1560$ arcsec, this equates to a total magnitude of

$$m_{\text{obs}} = -2.5 \log 2\pi r_0^2 + B(0) = 3.73$$

The total absolute magnitude of M31's disk, corrected for extinction, is then $M_B = -20.891$

The integration of bulge component is a bit trickier. If you start with

$$\log\left(\frac{I}{I_e}\right) = -3.33071 \left\{ \left(\frac{R}{R_e}\right)^{1/4} - 1 \right\} \implies (\log e) \ln\left(\frac{I}{I_e}\right) = -3.33071 \left\{ \left(\frac{R}{R_e}\right)^{1/4} - 1 \right\}$$

then

$$I(r) = I_e \, \mathsf{exp} \left\{ -7.67 \left(\frac{R}{R_e} \right)^{1/4} + 7.67 \right\} = I_e e^{7.67} \, \mathsf{exp} \left\{ -7.67 \left(\frac{R}{R_e} \right)^{1/4} \right\}$$

and the integral becomes

$$2\pi I_e \, e^{7.67} \int_0^\infty e^{-7.67 R_e^{-1/4}} e^{-7.67 R} R \, dR$$

If you let $R = x^4$, then the integral is analytic of the form

$$\int_0^\infty x^7 e^{-ax} dx = \frac{7!}{a^8}$$

Integrating this yields an apparnt bulge magnitude of $m_{\mathsf{obs}} = 4.365$, and an extinction-corrected absolute magnitude of -20.264. According to these numbers, the disk of M31 contains more B light than M31's bulge.